

# HIGHLIGHTS IN THE ANALYSIS OF EXCLUSIVE $B$ DECAYS

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## Abstract

I briefly describe recent developments in the theoretical analysis of non leptonic and semileptonic  $B$  decays. For non leptonic transitions, I focus on factorization, from the naive formulation to the most recent achievements. As for semileptonic decays, I mainly consider  $B$  transitions to excited charmed states.

## 1 Non Leptonic $B$ Meson Transitions

The theoretical description of non leptonic decays is very difficult since the final state is composed only of hadrons, thus requiring the consideration of the interplay between weak and strong dynamics. This is achieved using an Operator Product Expansion to write the effective hamiltonian describing a given weak decay as a sum of local operators, weighted by Wilson coefficients, both depending on a scale  $\mu$ . The coefficients include short distance dynamics and hence can be computed perturbatively, while the matrix elements of the operators include long distance physics at scales below  $\mu$ , in such a way that the dependence on the scale cancels in the product. A simple approach to evaluate such matrix elements is the "naive" factorization [1]. Let us consider the process  $\bar{B}^0 \rightarrow D^+ \pi^-$ : the effective hamiltonian is  $H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [C_1(\mu) O_1 + C_2(\mu) O_2]$ , where  $O_1 = (\bar{c}b)_{V-A} (\bar{d}u)_{V-A}$ ,  $O_2 = (\bar{c}u)_{V-A} (\bar{d}b)_{V-A}$ , and  $(\bar{q}_1 q_2)_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$ . Using the properties of the Gell-Mann matrices  $T^a$ , one has  $O_2 = \frac{O_1}{N_C} + 2\tilde{O}$ , giving  $H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [a_1(\mu) O_1 + 2C_2(\mu) \tilde{O}]$ , with  $a_1(\mu) = C_1(\mu) + C_2(\mu)/N_C$  and  $\tilde{O} = (\bar{c} T^a b)_{V-A} (\bar{d} T^a u)_{V-A}$ . Naive factorization consists in factorizing the matrix element  $\langle \pi^- D^+ | O_1 | \bar{B}^0 \rangle$  and neglecting the contribution of  $\tilde{O}$ . This is because

of the "colour transparency", an argument due to Bjorken according to which the quarks of the emitted pion move fast away from the interaction region and behave as a colour singlet, with no coupling to the coloured currents. The final amplitude reads as  $\mathcal{A}(\bar{B}^0 \rightarrow D^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1(\mu) \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle \langle D^+ | (\bar{c}b)_{V-A} | \bar{B}^0 \rangle$  and can be expressed in terms of  $B \rightarrow D$  form factors and  $f_\pi$ . The decays taking contribution only from operators like  $O_1$  are referred to as Class I decays; those taking contribution only from  $O_2$  as Class II and those taking both contributions as Class III ones. Since the matrix elements in the factorized expression do not depend on  $\mu$  any more, the scale dependence of the  $a_i$  is not matched, therefore naive factorization cannot be exact.

An improvement is represented by generalized factorization [2], describing non factorizable terms through parameters assumed universal for processes with similar kinematics. For example, one can write  $a_1 = C_1 + \xi C_2$ , and consider  $\xi = 1/N_C$  as a free parameter to be experimentally fitted. Another possibility is to split a non leptonic amplitude in substructures which can either be classified according to their transformation properties under  $SU(3)_F$  or associated to Wick contractions of the operators in the effective hamiltonian [3]. Symmetry arguments allow to derive relations or to establish hierarchies among such substructures.

A recent study [4] has derived a QCD factorization formula, which can be applied to many, but not all,  $B$  decays in the limit  $m_b \rightarrow \infty$ <sup>1</sup>. This analysis demonstrates that in the decay  $B \rightarrow M_1 M_2$ , non factorizable contributions are due to hard gluon exchange, while soft effects are confined to the system  $(B M_1)$ , if  $M_1$  is the meson which picks up the spectator quark of the  $B$ . Naive factorization is recovered at leading order in  $\Lambda_{QCD}/m_b$  and  $\alpha_s$ . The QCD factorization formula is then expressed in terms of form factors describing the transitions  $B \rightarrow M_1, M_2$  and the light cone wave functions of the particles (we refer to [4] for the explicit formula). It is proven that non factorizable topologies (vertex corrections, penguin diagrams, hard spectator interactions and annihilation diagrams) are free from infrared divergences at leading order in  $\Lambda_{QCD}/m_b$ . They represent  $\mathcal{O}(\alpha_s)$  contributions which are non universal, depending on the meson  $M_2$  which does not pick up the spectator quark. Finally, the absence of infrared divergences cannot be proven if  $M_2$  is a heavy meson.

The role of subleading terms in  $1/m_b$  remains an open question; it could be sizable if the leading term is suppressed for some reasons (colour suppression, CKM suppression, small Wilson coefficients). This could be the case for  $B \rightarrow K\pi$  [6]. The quantitative estimate of such terms would be fundamental in many respects; in particular, it would put the extraction of CP violating parameters from several non leptonic decay modes on a firmer theoretical basis.

## 2 Exclusive Semileptonic $b \rightarrow c$ Processes

In the limit  $m_Q \rightarrow \infty$ , where  $Q$  is a heavy quark with  $m_Q \gg \Lambda_{QCD}$ , Heavy Quark Effective Theory (HQET) exploits the decoupling of the light degrees of freedom to classify the heavy mesons in doublets [7]. The members of these differ only for the orientation of the heavy quark spin with respect to the angular momentum of the light degrees of freedom  $\vec{s}_\ell = \vec{\ell} + \vec{s}_q$ ,  $\vec{\ell}$  being the orbital angular momentum and  $\vec{s}_q$  the light quark spin. The low-lying doublet  $(P, P^*)$ , with

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<sup>1</sup>An alternative approach has been proposed in [5].

$P = D, B$ , corresponds to  $\ell = 0$  and has spin-parity  $J^P = (0^-, 1^-)$ . For  $\ell = 1$ , the two doublets  $(P_0, P'_1)$  and  $(P_1, P_2)$  have  $J^P = (0^+, 1^+)$  and  $J^P = (1^+, 2^+)$ , respectively. Excited heavy mesons have been identified in the charm sector and observed also in the beauty case [8]. The transitions between the members of two doublets are all described in terms of a universal function: the transitions  $(P, P^*) \rightarrow (P, P^*)$  are described by the Isgur-Wise function  $\xi(y)$ , where  $y = v \cdot v'$  and  $v$  and  $v'$  are the four velocities of the heavy meson in the initial and final state, respectively.  $\xi$  takes the place of 6 form factors and is normalized to 1 in the zero recoil point. The inclusion of short distance corrections, as well as the Luke theorem [9], assuring the absence of  $1/m_b$  corrections in some circumstances, lead to rather precise theoretical predictions for the  $B \rightarrow D^*$  semileptonic decay. The comparison with the data allows the most accurate determination of  $V_{cb}$  [10]. Moreover,  $(P, P^*) \rightarrow (P_0, P'_1)$  transitions involve the universal function  $\tau_{1/2}(y)$ , while  $(P, P^*) \rightarrow (P_1, P_2)$  involve the function  $\tau_{3/2}(y)$ ; the two  $\tau$  functions take the place of 14 form factors.

It is interesting to understand the contribution of semileptonic  $B$  decays to excited charmed mesons to the inclusive semileptonic  $B$  branching ratio. Moreover, the calculation of the  $\tau$  functions is worth due to their universality within HQET and their role in the Bjorken and Voloshin sum rules [11]. In general, HQET does not allow the determination of these universal functions, and, in particular, does not predict their normalization at zero recoil: non perturbative techniques are required. QCD sum rules [12] have been widely applied to this aim <sup>2</sup>. The function  $\tau_{1/2}$  has been computed including  $\mathcal{O}(\alpha_s)$  corrections [14], which are moderate as for the  $\xi$  function [15], while  $\tau_{3/2}$  has been estimated at leading order in  $\alpha_s$  [16]. The prediction for the semileptonic branching ratios:

$$\begin{aligned} BR(B \rightarrow D_0 \ell \nu_\ell) &= (5 \pm 3) \times 10^{-4} & BR(B \rightarrow D'_1 \ell \nu_\ell) &= (7 \pm 5) \times 10^{-4} \\ BR(B \rightarrow D_1 \ell \nu_\ell) &= 3.2 \times 10^{-3} & BR(B \rightarrow D_2 \ell \nu_\ell) &= 4.8 \times 10^{-3} \end{aligned} \quad (1)$$

must be compared to the data [8]:  $BR(B^+ \rightarrow \bar{D}_1^0(2420)\ell^+\nu_\ell) = (5.6 \pm 1.6) \times 10^{-3}$ ,  $BR(B^+ \rightarrow \bar{D}_2^{*0}(2460)\ell^+\nu_\ell) < 8 \times 10^{-3}$ , though the  $\bar{D}_1^0(2420)$  is likely to be an admixture of the two  $1^+$  states. The method predicts also the masses of the excited states, with the result:  $\bar{\Lambda}_{1/2}^+ = M_{D_0} - m_c = 1.0 \pm 0.1$  GeV,  $\bar{\Lambda}_{3/2}^+ = M_{D_1} - m_c = 1.05 \pm 0.1$  GeV, in agreement with existing data.

Higher states can be further considered. If  $l = 2$ , two doublets  $(D_1^*, D_2^*)$  and  $(D_2^*, D_3^*)$  are found with  $J_{s_\ell}^P = (1^-, 2^-)_{3/2}$  and  $J_{s_\ell}^P = (2^-, 3^-)_{5/2}$  respectively. QCD sum rules predict a very small universal function for  $B \rightarrow (D_1^*, D_2^*)$  decays, so that the corresponding transitions are negligible [17]. However, the transitions to the states belonging to the doublet with  $s_\ell = 5/2$  are presumably observable, with predicted branching ratios  $BR(B \rightarrow D_2^{*} \ell \nu_\ell) \simeq BR(B \rightarrow D_3^{*} \ell \nu_\ell) \simeq 1 \times 10^{-5}$  [17]. Since their strong decays proceed through F-wave transitions, we expect these states to be rather narrow, which is confirmed by the estimate of their width obtained varying their strong coupling to pions in the range [0.2, 0.5] [17].

Also in this case it is important to analyse the role of  $\mathcal{O}(\frac{1}{m_b})$  corrections, when the contribution of higher dimensional operators in the effective currents and in the effective HQET lagrangian should be considered [18, 19, 20]. For the semileptonic transition  $B \rightarrow D_2 \ell \nu_\ell$  these corrections are rather modest, while they

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<sup>2</sup>A comparison with the results of other approaches, when available, is provided in [13].

turn out to be large for  $B \rightarrow D_1 \ell \nu_\ell$  [20]. Further improvements in the analysis of such corrections, including next-to-leading order QCD terms, are required.

### 3 Conclusions

Important theoretical progresses have been achieved in the study of exclusive  $B$  decays, mainly exploiting the large value of  $m_b$ . Open questions remain, such as the role and the classification of  $1/m_b$  corrections. New theoretical efforts are therefore required to match the experimental accuracy expected in next few years.

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